1. Sean $A \in \mathbb{R}^{2\times 4}$ y $B \in \mathbb{R}^{4\times 3}$ las matrices definidas por

$$A = \begin{bmatrix} 0 & 1 & 1 & 0 \\ 1 & 1 & 0 & 1 \end{bmatrix} \text{ y } B = \begin{bmatrix} 1 & 1 & 2 \\ 1 & 0 & 1 \\ 1 & 2 & 3 \\ 0 & 1 & 1 \end{bmatrix}.$$

Hallar una base del subespacio $\mathrm{nul}(A) \cap \mathrm{col}(B)$.

$\mathrm{nul}(A) \subset \mathbb{R}^4$ $\quad$ $\mathrm{Col}(B) \subset \mathbb{R}^4$ $\quad$ A tiene dos filas $\{F_1, F_2\}$ LI

$\dim \mathrm{nul}(A) + \mathrm{rg}(A) = 4 \Rightarrow \dim \mathrm{nul}(A) = 2$

Si miramos las columnas de B $\quad C_1 + C_2 = C_3$

$\dim \mathrm{Col}(B) = 2 \qquad 0 \leq \dim\left(\mathrm{nul}(A) \cap \mathrm{Col}(B)\right) \leq 2$

$\mathrm{nul}(A) = \{v \in \mathbb{R}^4 : x_2 + x_3 = 0 \ \wedge \ x_1 + x_2 + x_4 = 0\} \quad (Av = 0)$

Si $x \in \mathrm{Col}(B) \Rightarrow x = \alpha \begin{pmatrix} 1 \\ 1 \\ 1 \\ 0 \end{pmatrix} + \beta \begin{pmatrix} 1 \\ 0 \\ 2 \\ 1 \end{pmatrix} = \begin{pmatrix} \alpha + \beta \\ \alpha \\ \alpha + 2\beta \\ \beta \end{pmatrix}$ como quiero

que $x \in \mathrm{nul}(A) \Rightarrow \overset{x_2}{\alpha} + \overset{x_3}{\alpha + 2\beta} = 0$

$\alpha = -\beta$

$\overset{x_1}{\alpha + \beta} + \overset{x_2}{\alpha} + \overset{x_4}{\beta} = 0 \quad \alpha = -\beta$

$\Rightarrow \quad x \in \mathrm{Nul}(A) \cap \mathrm{Col}(B) \quad$ si $\quad x = \alpha \begin{pmatrix} 1 \\ 1 \\ 1 \\ 0 \end{pmatrix} = \alpha \begin{pmatrix} 1 \\ 0 \\ 2 \\ 1 \end{pmatrix} = \alpha \begin{pmatrix} 0 \\ 1 \\ -1 \\ -1 \end{pmatrix}$

$\Rightarrow \quad \mathrm{Nul}(A) \cap \mathrm{Col}(B) = \mathrm{gen}\left\{ \begin{pmatrix} 0 \\ 1 \\ -1 \\ -1 \end{pmatrix} \right\}$

QED la Rta : una base di $\mathrm{Nul}(A) \cap \mathrm{Col}(B) = \left\{ \begin{pmatrix} 0 \\ 1 \\ -1 \\ -1 \end{pmatrix} \right\}$

$\mathrm{Nul}(A) + \mathrm{Col}(B) = S \qquad \underline{\dim S} = \overbrace{\dim \mathrm{Nul}(A)}^{2} + \overbrace{\dim \mathrm{Col}(B)}^{2} - \underline{\dim \mathrm{Nul}(A) \cap \mathrm{Col}(B)}$

$2 \leq * \leq 4 \qquad\qquad 0 \leq * \leq 2$

2. Sea $T : \mathbb{R}^{2\times 2} \to \mathbb{R}^3$ la transformación lineal definida por:

$\dim \operatorname{nul}(B) + \dim \operatorname{Col}(B) = 4$

$\Rightarrow \dim \operatorname{nul}(B) = 2$

$$B = [T]_{\mathcal{B}}^{\mathcal{E}} = \begin{bmatrix} 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 \end{bmatrix}$$

$\operatorname{rg}(B) = 2$

$C_3 = C_4 = C_1 + C_2$

$\{C_1, C_2\}$ LI

donde $\mathcal{E}$ es la base canónica de $\mathbb{R}^3$ y $\mathcal{B}$ es la base de $\mathbb{R}^{2\times 2}$ definidas por:

$$\mathcal{B} = \left\{ \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}, \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} \right\}.$$

$T^{-1}\begin{pmatrix} 6 \\ 6 \\ 6 \end{pmatrix}$

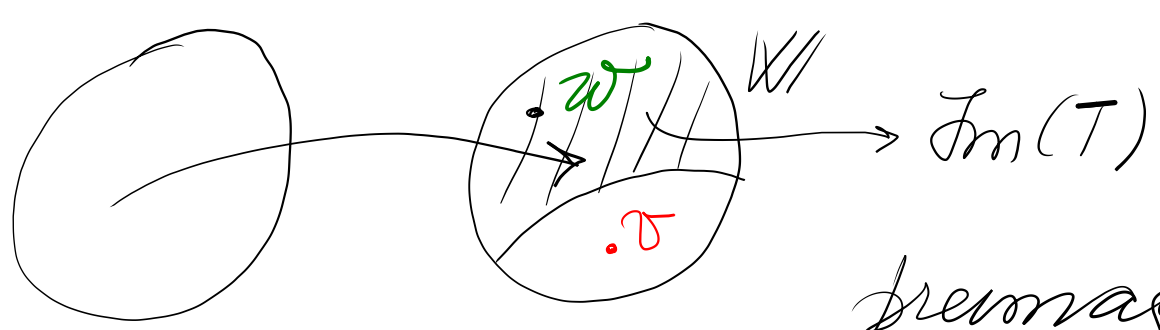
Hallar el conjunto de todas las $A \in \mathbb{R}^{2\times 2}$ tales que $T(A) = [6 \quad 6 \quad 6]$.

Teo dim

$$\dim(\mathbb{R}^{2\times 2}) = 4 = \underbrace{\dim \operatorname{nul}(T)}_{\geq 1} + \underbrace{\dim \operatorname{Im}(T)}_{\leq 3}$$

$\operatorname{Im}(T) \subset \mathbb{R}^3$

$\Rightarrow$ hay infinitas soluciones pues la T no es inyectiva



preimagen de $v \longrightarrow \emptyset$

preimagen de w al menos hay una $\begin{cases} \text{una sola si T es inyectiva} \\ \text{infinitas si no lo es} \end{cases}$

$$\underbrace{[T]_B^E}\,\underbrace{[A]^B}_{[T(A)]^E = T(A)} = \begin{pmatrix} 6 \\ 6 \\ 6 \end{pmatrix}$$

$$[A]^B = \begin{pmatrix} a \\ b \\ c \\ d \end{pmatrix}$$

$$\mathcal{B} = \left\{ \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}, \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} \right\}$$

$$\begin{pmatrix} 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 \end{pmatrix} \begin{pmatrix} a \\ b \\ c \\ d \end{pmatrix} = \begin{pmatrix} 6 \\ 6 \\ 6 \end{pmatrix}$$

$$\left(\begin{array}{cccc|c} 1 & 0 & 1 & 1 & 6 \\ 0 & 1 & 1 & 1 & 6 \\ 1 & 0 & 1 & 1 & 6 \end{array}\right) \begin{array}{l} \checkmark \\ \checkmark \\ F_3 - F_1 \end{array} \longrightarrow \left(\begin{array}{cccc|c} 1 & 0 & 1 & 1 & 6 \\ 0 & 1 & 1 & 1 & 6 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right)$$

S. Comp Indeterminado

$$b = 6 - c - d \qquad a = 6 - c - d$$

$$\Rightarrow [A]^B = \begin{pmatrix} 6-c-d \\ 6-c-d \\ c \\ d \end{pmatrix} \Rightarrow A = (6-c-d)\begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} + (6-c-d)\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} + c\begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} + d\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$$

$$A = \begin{pmatrix} 6 & 6 \\ 6 & 0 \end{pmatrix} + c\underbrace{\begin{pmatrix} -1 & -1 \\ -1 & 1 \end{pmatrix}} + d\underbrace{\begin{pmatrix} -1 & 0 \\ -2 & 0 \end{pmatrix}}_{\in \operatorname{Im}(T)}$$

Solución particular

$c, d \in \mathbb{R}$

controlar la cuenta.

3. Sea $\Sigma : \mathbb{R}^3 \to \mathbb{R}^3$ la simetría respecto del plano $\{x \in \mathbb{R}^3 : x_1 - x_2 = 0\}$ en la dirección de la recta generada por el vector $\begin{bmatrix}1 & -1 & 0\end{bmatrix}^T$. Hallar la imagen por Σ del triángulo de vértices $v_1 = \begin{bmatrix}0 & 0 & 0\end{bmatrix}^T$, $v_2 = \begin{bmatrix}0 & 0 & 1\end{bmatrix}^T$ y $v_3 = \begin{bmatrix}6 & -6 & 6\end{bmatrix}^T$.

$\Sigma \circ \Sigma = I_d$ (Simetría) $\qquad \Pi \circ \Pi = \Pi$ (proyección)

Resp plano $\{x \in \mathbb{R}^3 : x_1 - x_2 = 0\} = \text{gen}\left\{\begin{pmatrix}1\\1\\0\end{pmatrix}, \begin{pmatrix}0\\0\\1\end{pmatrix}\right\} = S_1$

$S_1 \oplus S_2 = \mathbb{R}^3$

$\Sigma\begin{pmatrix}1\\1\\0\end{pmatrix} = \begin{pmatrix}1\\1\\0\end{pmatrix} \qquad \Sigma\begin{pmatrix}0\\0\\1\end{pmatrix} = \begin{pmatrix}0\\0\\1\end{pmatrix}$

$S_2 = \text{gen}\left\{\begin{pmatrix}-1\\1\\0\end{pmatrix}\right\}$

en la dirección .... $\Sigma\begin{pmatrix}1\\-1\\0\end{pmatrix} = \begin{pmatrix}-1\\1\\0\end{pmatrix}$

$\Sigma(v_1) = v_1 = 0 \in S_1 \qquad \Sigma(v_2) = v_2 \in S_1 \qquad \Sigma_{S_1 S_2}$

$$\begin{pmatrix}6\\-6\\6\end{pmatrix} = \underbrace{\underset{0}{\alpha}\begin{pmatrix}1\\1\\0\end{pmatrix} + \underset{6}{\beta}\begin{pmatrix}0\\0\\1\end{pmatrix}}_{\in S_1} + \underbrace{\underset{6}{\gamma}\begin{pmatrix}1\\-1\\0\end{pmatrix}}_{\in S_2}$$

$\alpha + \gamma = 6$

$\alpha - \gamma = -6$

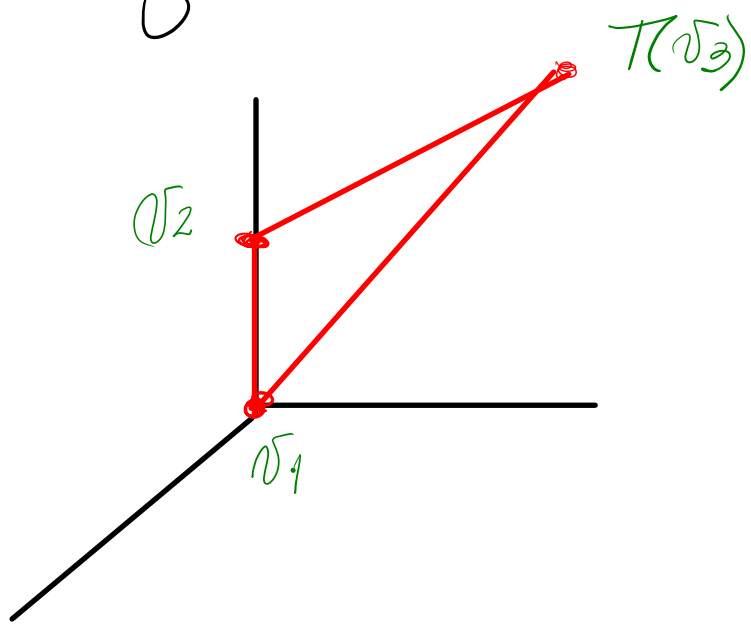
$\alpha = 0$

$\gamma = 6$

$$\Sigma\begin{pmatrix}6\\-6\\6\end{pmatrix} = 0\begin{pmatrix}1\\1\\0\end{pmatrix} + 6\begin{pmatrix}0\\0\\1\end{pmatrix} - 6\begin{pmatrix}1\\-1\\0\end{pmatrix} = \begin{pmatrix}-6\\6\\6\end{pmatrix}$$

$\begin{pmatrix}-6\\6\\6\end{pmatrix}$ $\begin{pmatrix}0\\0\\0\end{pmatrix}$ $\begin{pmatrix}0\\0\\1\end{pmatrix}$ no están alineados

$\Rightarrow$ la imagen del triáng de extremos v_1, v_2, v_3 es el triáng de extremos $v_1, v_2, \begin{pmatrix}-6\\6\\6\end{pmatrix}$



$$\Sigma\begin{pmatrix}1\\1\\0\end{pmatrix}=\begin{pmatrix}1\\1\\0\end{pmatrix} \quad \Sigma\begin{pmatrix}0\\0\\1\end{pmatrix}=\begin{pmatrix}0\\0\\1\end{pmatrix}$$

$$\Sigma\begin{pmatrix}1\\-1\\0\end{pmatrix}=\begin{pmatrix}-1\\1\\0\end{pmatrix}$$

$$\mathcal{B}_{\mathbb{R}^3}=\Big\{\underbrace{\begin{pmatrix}1\\1\\0\end{pmatrix}\begin{pmatrix}0\\0\\1\end{pmatrix}}_{\in S_1}\ \underbrace{\begin{pmatrix}1\\-1\\0\end{pmatrix}}_{\in S_2}\Big\}$$

$$\begin{pmatrix}x_1\\x_2\\x_3\end{pmatrix}=\underbrace{\alpha}_{\frac{x_1+x_2}{2}}\begin{pmatrix}1\\1\\0\end{pmatrix}+\underbrace{\beta}_{x_3}\begin{pmatrix}0\\0\\1\end{pmatrix}+\underbrace{\gamma}_{\frac{x_1-x_2}{2}}\begin{pmatrix}1\\-1\\0\end{pmatrix}$$

$$\alpha+\gamma=x_1$$
$$\alpha-\gamma=x_2$$

$$\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \underbrace{\alpha}_{\frac{x_1+x_2}{2}} \underbrace{\begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}}_{v_1} + \underbrace{\beta}_{x_3} \underbrace{\begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}}_{v_2} + \underbrace{\gamma}_{\frac{x_1-x_2}{2}} \underbrace{\begin{pmatrix} 1 \\ -1 \\ 0 \end{pmatrix}}_{v_3}$$

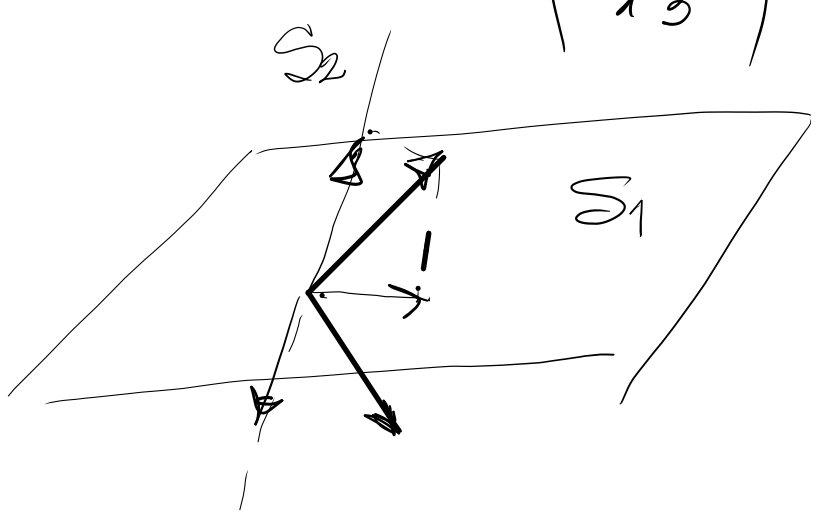
$$\mathcal{B} = \{v_1, v_2, v_3\}$$

$$[\Sigma]_{\mathcal{B}}^{\mathcal{B}} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{pmatrix}$$

$$\Sigma \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \frac{x_1+x_2}{2} \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} + x_3 \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} + \frac{x_1-x_2}{2} \begin{pmatrix} 1 \\ -1 \\ 0 \end{pmatrix}$$

$$= \begin{pmatrix} x_2 \\ x_1 \\ x_3 \end{pmatrix}$$

$$\Sigma \begin{pmatrix} 6 \\ -6 \\ 6 \end{pmatrix} = \begin{pmatrix} -6 \\ 6 \\ 6 \end{pmatrix} \checkmark$$



$$[\Sigma]_{E}^{E} = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

5. Se considera el espacio euclídeo $(\mathbb{R}_2[x], \langle\cdot,\cdot\rangle)$ con el producto interno definido por

$$\langle a_0 + a_1x + a_2x^2, b_0 + b_1x + b_2x^2\rangle = a_0b_0 + a_1b_1 + a_2b_2.$$

Sean $\mathbb{S} = \text{gen}\{\overset{p_1}{1 - 2x - 2x^2}, \overset{p_2}{2 + 2x - x^2}\}$ y $P_{\mathbb{S}} : \mathbb{R}_2[x] \to \mathbb{R}_2[x]$ la proyección ortogonal de $\mathbb{R}_2[x]$ sobre $\mathbb{S}$. Hallar todos los $q \in \mathbb{R}_2[x]$ tales que $P_{\mathbb{S}}(q) = 2 - 2x^2$ cuya distancia a $\mathbb{S}$ es 6.

$\dim S = 2$ $\qquad q \in \mathbb{R}_2[x]$ $\qquad q_S + q_{S^\perp} =$

$\dim S^\perp = 1$ $\qquad = P_S q + P_{S^\perp} q$

$S \oplus S^\perp = \mathbb{R}_2[x]$

$S^\perp = \{ p : p \perp p_1 \wedge p \perp p_2 \}$ $\qquad p = a + bx + cx^2$

$\langle a + bx + cx^2 ; 1 - 2x - 2x^2 \rangle = a - 2b - 2c = 0$ $\qquad 2a - 4b - 4c$

$\langle a + bx + cx^2 ; 2 + 2x - x^2 \rangle = 2a + 2b - c = 0$ $\qquad 2a + 2b - c$ $\qquad -6b - 3c = 0$

$\oplus \quad 3a - 3c = 0 \qquad a = c \qquad -2b = c$

$a = c \qquad b = -1/2\, c \qquad S^\perp = \text{gen}\{\overset{w}{2 - x + 2x^2}\}$ ✓

$\mathcal{B}_{\mathbb{R}_2[x]} = \{ \underbrace{1 - 2x - 2x^2 ; 2 + 2x - x^2}_{\in S} ; \underbrace{2 - x + 2x^2}_{\in S^\perp} \}$

$$q = \underbrace{2-2x^2}_{q_1 \in S} + \alpha\,(\underbrace{2-x+2x^2}_{q_2 \in S^\perp})$$

$$d(q;S) = 6$$

$$\Rightarrow \|P_{S^\perp} q\| = 6$$

$$q = q_1 + q_2$$

$$\alpha = \pm 2$$

$$\|q_2\| = 6 \qquad \alpha = \pm 2$$

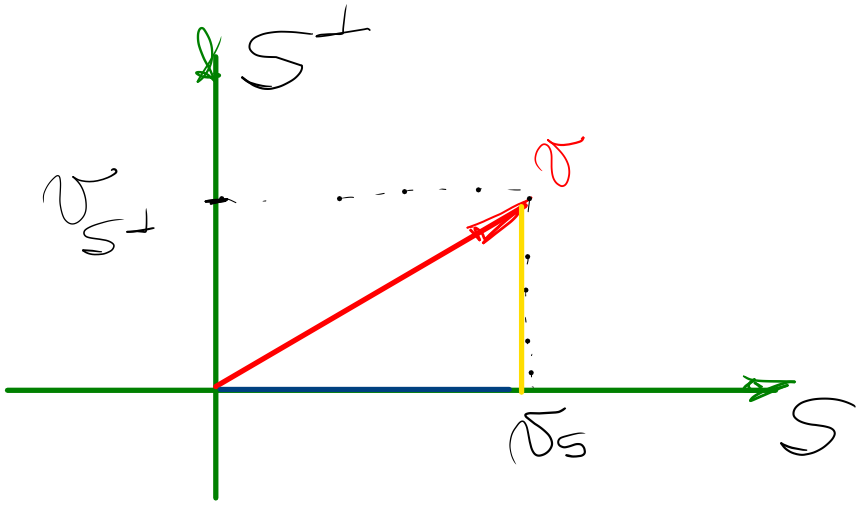
$$\|q_2\| = |\alpha|\,\|2-x+2x^2\| = |\alpha|\sqrt{2^2+(-1)^2+2^2} = |\alpha|\,3 = 6$$

$$\Rightarrow \alpha = \pm 2$$

$$2-2x^2 = \overset{2/3}{\alpha}\,(1-2x-2x^2) + \overset{2/3}{\beta}\,(2+2x-x^2)$$

$$\alpha + 2\beta = 2$$
$$-2\alpha + 2\beta = 0$$
$$\alpha = \beta = 2/3$$

Distance



$$d(v, S) = \|v_{S^\perp}\|$$

$$d(v, S^\perp) = \|v_S\|$$

$$d(v, 0) = \|v\|$$

$$\|v\|^2 = \|v_S\|^2 + \|v_{S^\perp}\|^2$$

$$\|v - 0\| = d(v, 0)$$

3) $\pi : \mathbb{R}^3 \to \mathbb{R}^3$ una TL que cumple $\pi \circ \pi = \pi$

Proyección

$\pi\begin{pmatrix}1\\2\\1\end{pmatrix} = \begin{pmatrix}1\\1\\0\end{pmatrix}$, $\pi\begin{pmatrix}0\\1\\2\end{pmatrix} = \begin{pmatrix}0\\0\\1\end{pmatrix}$

Hallar $\pi(S)$

$S = \{x \in \mathbb{R}^3 : x_1 = 0\}$

$\pi\begin{pmatrix}1\\1\\0\end{pmatrix} = \begin{pmatrix}1\\1\\0\end{pmatrix}$ $\pi\begin{pmatrix}0\\0\\1\end{pmatrix} = \begin{pmatrix}0\\0\\1\end{pmatrix}$

$S = \text{gen}\left\{\begin{pmatrix}0\\0\\1\end{pmatrix}, \begin{pmatrix}0\\1\\0\end{pmatrix}\right\}$

$\pi_{S_1 S_2}(v)$, $v \in S_1 \Rightarrow \pi(v) = v$
$v \in S_2 \Rightarrow \pi(v) = 0$

$T(S) = \left\{\begin{pmatrix}0\\0\\1\end{pmatrix}, \begin{pmatrix}0\\0\\-1\end{pmatrix}\right.$

$S_1 = \text{gen}\left\{\begin{pmatrix}1\\1\\0\end{pmatrix}, \begin{pmatrix}0\\0\\1\end{pmatrix}\right\} \to \dim(S_1) = 2 \Rightarrow \dim(S_2) = 1$

$\begin{pmatrix}0\\1\\2\end{pmatrix} = v_{S_2} + \begin{pmatrix}0\\0\\1\end{pmatrix} \Rightarrow v_{S_2} = \begin{pmatrix}0\\1\\1\end{pmatrix}$

$\begin{pmatrix}2\\1\\1\end{pmatrix} = w_{S_2} + \begin{pmatrix}1\\1\\0\end{pmatrix}$

$w_{S_2} = \begin{pmatrix}0\\1\\1\end{pmatrix}$

$S_2 = \text{gen}\left\{\begin{pmatrix}0\\1\\1\end{pmatrix}\right\}$

$S = \text{gen}\left\{\underbrace{\begin{pmatrix}0\\1\\1\end{pmatrix}}_{\in S_2} \quad \overbrace{\begin{pmatrix}0\\0\\1\end{pmatrix}}^{\in S_1}\right\}$

$\forall v \in S \Rightarrow$

$v = \alpha\begin{pmatrix}0\\1\\1\end{pmatrix} + \beta\begin{pmatrix}0\\0\\1\end{pmatrix} \Rightarrow$

$\forall v \in S \Rightarrow$

$$v = \alpha \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix} + \beta \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$$

$$\pi(v) = \alpha \, 0 + \beta \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$$

$$\pi(S) = \text{gen}\left\{ \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \right\}$$

$$\mathcal{B}_S = \left\{ \begin{pmatrix} 0 \\ 1 \\ 2 \end{pmatrix} \begin{pmatrix} 0 \\ 1 \\ -1 \end{pmatrix} \right\}$$

$v \in S \quad v = \alpha \begin{pmatrix} 0 \\ 1 \\ 2 \end{pmatrix} + \beta \begin{pmatrix} 0 \\ 1 \\ -1 \end{pmatrix}$

$$\begin{pmatrix} 0 \\ 1 \\ 2 \end{pmatrix} = \overset{1}{\alpha} \overset{\in S_2}{\begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix}} + \overset{1}{\beta} \overset{\in S_1}{\begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}} + \overset{0}{\gamma} \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}$$

$$\begin{pmatrix} 0 \\ 1 \\ 2 \end{pmatrix} \xrightarrow{\pi} \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$$

$$\begin{pmatrix} 0 \\ 1 \\ -1 \end{pmatrix} \to \begin{pmatrix} 0 \\ 0 \\ -2 \end{pmatrix}$$

$$\alpha \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} + \beta \begin{pmatrix} 0 \\ 0 \\ -2 \end{pmatrix}$$

$$\begin{pmatrix} 0 \\ 1 \\ -1 \end{pmatrix} = \overset{1}{\alpha} \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix} + \overset{-2}{\beta} \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} + \overset{0}{\gamma} \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}$$

$\Rightarrow \forall v \in S \quad \pi(v) = a \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$

2. Sea $T : \mathbb{R}_2[x] \to \mathbb{R}^3$ la transformación lineal definida por

$$[T]_{\mathcal{B}}^{\mathcal{E}} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 0 & -2 & 2 \end{bmatrix}, \quad \overline{F_1} = \overline{F_2} \; !$$

donde $\mathcal{E}$ es la base canónica de $\mathbb{R}^3$ y $\mathcal{B}$ es la base de $\mathbb{R}_2[x]$ definida por

$$\mathcal{B} = \left\{ x^2 - x,\ x^2 - 2x,\ x^2 - 3x + 2 \right\}.$$

$\frac{1}{2}(C_3 - C_1)$

Hallar la preimagen por T del subespacio $\mathbb{S} = \text{gen}\left\{ \begin{bmatrix} 6 & 6 & 6 \end{bmatrix}^T \right\}$.

$\mathbb{S}$

dim $\mathbb{R}_2[x] = 3$ $\quad$ rg $[T]_{\mathcal{B}}^{\mathcal{E}} = 2$ $\quad$ Im$(T) = \text{gen}\left\{ \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \right\}$

$$\begin{pmatrix} 6 \\ 6 \\ 6 \end{pmatrix} \in \text{Im}(T)$$

$$[T]_{\mathcal{B}}^{\mathcal{E}} [\phi]^{\mathcal{B}} = [T(\phi)]^{\mathcal{E}}$$

$$[\phi]^{\mathcal{B}} = \begin{pmatrix} a \\ b \\ c \end{pmatrix}$$

$$\left(\begin{array}{ccc|c} 1 & 1 & 1 & 6\alpha \\ 1 & 1 & 1 & 6\alpha \\ 0 & -2 & 2 & 6\alpha \end{array}\right)$$

$$-2b + 2c = 6\alpha$$
$$c = 3\alpha + b$$

$$a + b + c = 6\alpha$$
$$a + b + 3\alpha + b = 6\alpha$$
$$a = 3\alpha - 2b$$

$$[\phi]^{\mathcal{B}} = \begin{pmatrix} 3\alpha - 2b \\ b \\ 3\alpha + b \end{pmatrix} \Rightarrow$$

$$\phi = (3\alpha - 2b)(x^2 - x) + b(x^2 - 2x) + (3\alpha + b)(x^2 - 3x + 2)$$
$$= \ldots$$

terminar la cuenta

4. Se considera el espacio euclídeo $(\mathbb{R}^3, \langle\cdot,\cdot\rangle)$ con el producto interno definido por

$$\langle x, y\rangle = y^T \begin{bmatrix} 2 & 0 & 1 \\ 0 & 3 & 0 \\ 1 & 0 & 4 \end{bmatrix} x.$$

Sea $\mathbb{S} = \text{gen}\left\{\begin{bmatrix}1 & 1 & 1\end{bmatrix}^T\right\}$. Hallar $v_1 \in \mathbb{S}$ y $v_2 \in \mathbb{S}^\perp$ tales que $v_1 + v_2 = \begin{bmatrix}2 & 0 & 3\end{bmatrix}^T$.

$$S = \text{gen}\left\{\begin{pmatrix}1\\1\\1\end{pmatrix}\right\} \qquad S^\perp = \text{gen}\{u; w\} = \left\{\begin{pmatrix}x_1\\x_2\\x_3\end{pmatrix} \in \mathbb{R}^3 : 3x_1 + 3x_2 + 5x_3 = 0\right\}$$

$$\left\langle u; \begin{pmatrix}1\\1\\1\end{pmatrix}\right\rangle = \left\langle w; \begin{pmatrix}1\\1\\1\end{pmatrix}\right\rangle = 0$$

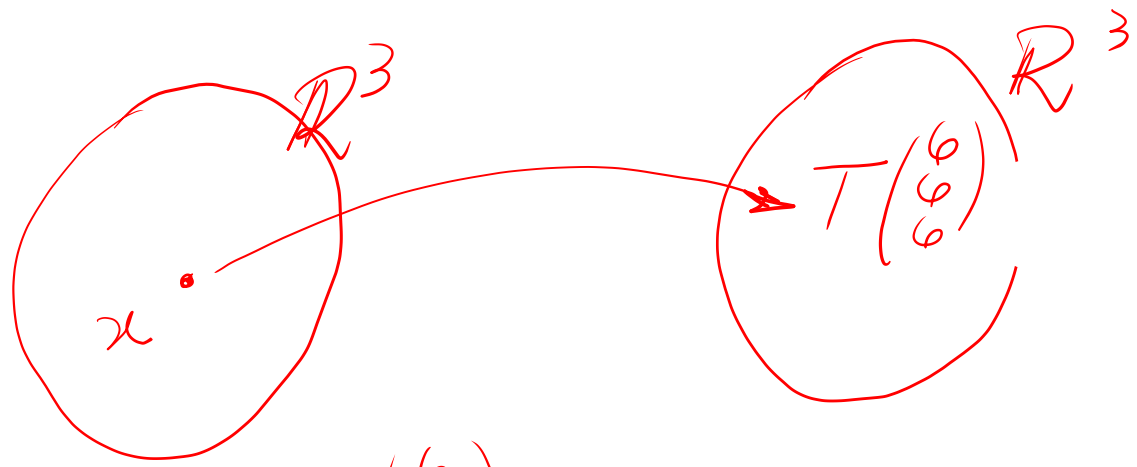
$$\left\langle u; \begin{pmatrix}1\\1\\1\end{pmatrix}\right\rangle = \begin{pmatrix}1 & 1 & 1\end{pmatrix} G \begin{pmatrix}x_1\\x_2\\x_3\end{pmatrix} = \overset{u}{\begin{pmatrix}3 & 3 & 5\end{pmatrix}} \overset{w}{\begin{pmatrix}x_1\\x_2\\x_3\end{pmatrix}} = 0$$

$$v_1 + v_2 = \begin{pmatrix}2\\0\\3\end{pmatrix} = \underbrace{\alpha\begin{pmatrix}1\\1\\1\end{pmatrix}}_{\in S} + \underbrace{\beta\begin{pmatrix}5\\0\\-3\end{pmatrix} + \gamma\begin{pmatrix}0\\5\\-3\end{pmatrix}}_{\in S^\perp}$$

3. Sea $T : \mathbb{R}^3 \to \mathbb{R}^3$ la transformación lineal definida por

$$[T]_{\mathcal{B}}^{\mathcal{E}} = \begin{bmatrix} 1 & 1 & 0 \\ 3 & 1 & -2 \\ 0 & 1 & 1 \end{bmatrix},$$

donde $\mathcal{B}$ es la base de $\mathbb{R}^3$ definida por $\mathcal{B} = \left\{ \begin{bmatrix} 1 & 0 & 0 \end{bmatrix}^T, \begin{bmatrix} 1 & 0 & 1 \end{bmatrix}^T, \begin{bmatrix} 0 & 1 & 1 \end{bmatrix}^T \right\}$ y $\mathcal{E}$ es la base canónica de $\mathbb{R}^3$. Hallar el conjunto de todos los $x \in \mathbb{R}^3$ tales que $T(x) = T\left(\begin{bmatrix} 6 & 6 & 6 \end{bmatrix}^T \right)$.



$x = \begin{pmatrix} 6 \\ 6 \\ 6 \end{pmatrix}$ particular

TODAS las soluciones son $\begin{pmatrix} 6 \\ 6 \\ 6 \end{pmatrix} + \alpha v$, $v \in \operatorname{Im}(T)$

$$[T]_{\mathcal{B}}^{\mathcal{E}} [v]^{\mathcal{B}} = \begin{pmatrix} 1 & 1 & 0 \\ 3 & 1 & -2 \\ 0 & 1 & 1 \end{pmatrix} \underbrace{\begin{pmatrix} a \\ b \\ c \end{pmatrix}} = 0$$

$$a = -b = c$$

$$\operatorname{Im}(T) = \operatorname{gen}\left\{ a \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} - a \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix} + a \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix} = a \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} \right\}$$

$$\Rightarrow \quad x = \begin{pmatrix} 6 \\ 6 \\ 6 \end{pmatrix} + a \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$$