1 $A \in \mathbb{R}^{3\times 3}$ $\quad Fil(A) = \{v \in \mathbb{R}^3 \; x_1 - x_3 = 0\} = gen\left\{ \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix} \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} \right\}$

$\in Col(A) \in Col(A)$

$$A \begin{pmatrix} 1 & 1 \\ 0 & 1 \\ 1 & 1 \end{pmatrix} = \begin{pmatrix} -1 & 2 \\ 2 & -1 \\ 2 & 2 \end{pmatrix} \qquad \mathcal{B}_{\mathbb{R}^3} =$$

son CL de las columnas de A

como $\left\{ \begin{pmatrix} -1 \\ 2 \\ 2 \end{pmatrix} \begin{pmatrix} 2 \\ -1 \\ 2 \end{pmatrix} \right\}$ LI y $\dim Col(A) = \dim Fil(A) = 2$

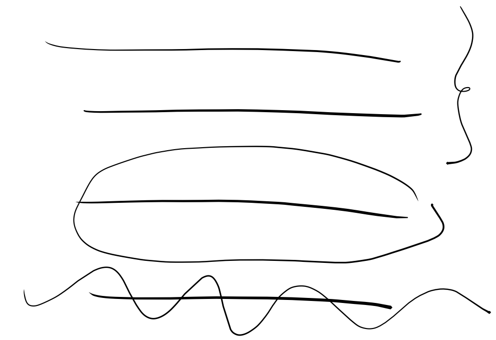
$\Rightarrow \left\{ \begin{pmatrix} -1 \\ 2 \\ 2 \end{pmatrix} \begin{pmatrix} 2 \\ -1 \\ 2 \end{pmatrix} \right\}$ es base de $Col(A)$

$$A \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix} = \begin{pmatrix} Fil_1(A) \\ Fil_2(A) \\ Fil_3(A) \end{pmatrix} \overset{v}{\begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}} = \begin{pmatrix} \langle F_1(A); v \rangle \\ \langle F_2(A); v \rangle \\ \langle F_3(A); v \rangle \end{pmatrix}$$

$$A \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix} = \begin{pmatrix} C_1(A) & C_2(A) & C_3(A) \end{pmatrix} \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix} = (C_1 + C_3)(A)$$

$$\mathcal{B}_{\mathbb{R}^3} = \left\{ \begin{pmatrix} -1 \\ 2 \\ 2 \end{pmatrix}, \begin{pmatrix} w \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} \right.$$

$\mathrm{Col}(A)$ $\quad$ $\mathrm{Fil}(A)$



$$w \in \mathrm{Col}(A) \cap \mathrm{Fil}(A)$$

$$\underbrace{\left\{ \begin{pmatrix} -1 \\ 2 \\ 2 \end{pmatrix} \overset{w}{\begin{pmatrix} 2 \\ -1 \\ 2 \end{pmatrix}} \right\}}_{\in \mathrm{Fil}(A)}$$

Si $w \in \mathrm{Col}(A) \Rightarrow$

$$w = \alpha \begin{pmatrix} -1 \\ 2 \\ 2 \end{pmatrix} + \beta \begin{pmatrix} 2 \\ -1 \\ 2 \end{pmatrix} = \begin{pmatrix} -\alpha + 2\beta \\ 2\alpha - \beta \\ 2\alpha + 2\beta \end{pmatrix}$$

Condición $x_1 = x_3 \Rightarrow -\alpha + \cancel{2\beta} = 2\alpha + \cancel{2\beta}$

$$\alpha = 0$$

(1) $\mathcal{B}_1$ no se usa

$$M_{\mathcal{B}_1}^{\mathcal{B}_2} [v]^{\mathcal{B}_1} = [v]^{\mathcal{B}_2}$$

dato ? dato

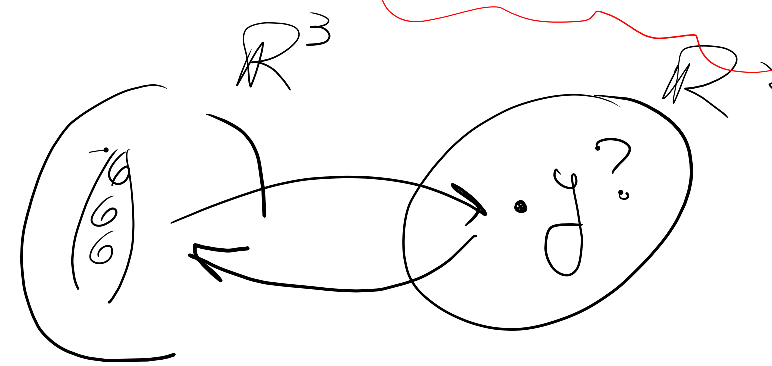
$$\begin{pmatrix} 1 & 3 & 2 \\ 3 & 2 & 1 \\ 2 & 1 & 3 \end{pmatrix} \begin{pmatrix} a \\ b \\ c \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$$

$$\begin{pmatrix} a \\ b \\ c \end{pmatrix} = \left(M_{\mathcal{B}_1}^{\mathcal{B}_2} \right)^{-1} \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$$

$$[v]^{\mathcal{B}_1} = 1/18 \begin{pmatrix} -5 \\ 7 \\ 1 \end{pmatrix}$$

(3) $\mathbb{R}^3 \longrightarrow \mathbb{R}^3$

$x \in \mathbb{R}^3 : T(x) = T\begin{pmatrix} 6 \\ 6 \\ 6 \end{pmatrix}$



$$[T]_B^E \, [v]^B = [T(v)]^E$$

dato

$\begin{pmatrix} 6 \\ 6 \\ 6 \end{pmatrix}$ es una solución particular

$$[T]_B^E \, [v]^B = [0_V]^E = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

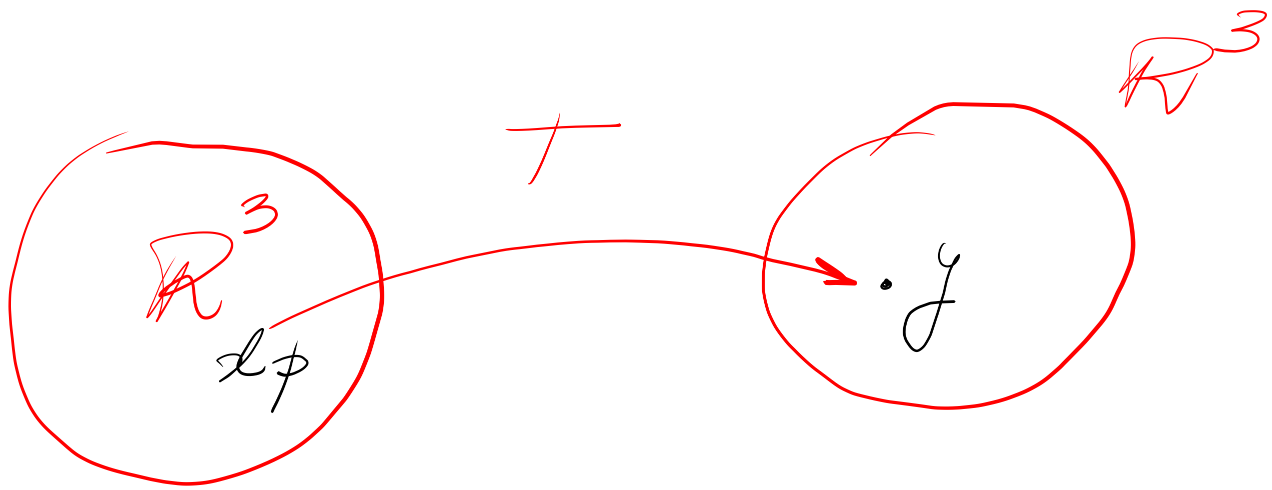
termino Trán

$$[T]_B^E \, [v]^B = ?$$

$$[T]_B^E \, [v]^B = y$$

$$[T]_B^E \left[\begin{pmatrix} 6 \\ 6 \\ 0 \end{pmatrix}\right]^B = \left[T\begin{pmatrix} 6 \\ 6 \\ 6 \end{pmatrix}\right]^E$$

$$[T]_B^E \begin{pmatrix} 6 \\ 0 \\ 6 \end{pmatrix} = \begin{pmatrix} y \end{pmatrix}$$



Halla

x :

$$T(x) = T\begin{pmatrix}6\\6\\6\end{pmatrix} = y$$

Si fuera inyectiva la solución es $x_p = \begin{pmatrix}6\\6\\6\end{pmatrix}$

T no es inyectiva las soluciones $x = x_p + \alpha v$, $v \in Nu(T)$

$$T(v) = 0_{\mathbb{R}^3}$$

$$[T]_B^E [v]^B = [T(v)]^E = \begin{pmatrix}0\\0\\0\end{pmatrix} \qquad B = \{v_1, v_2, v_3\}$$

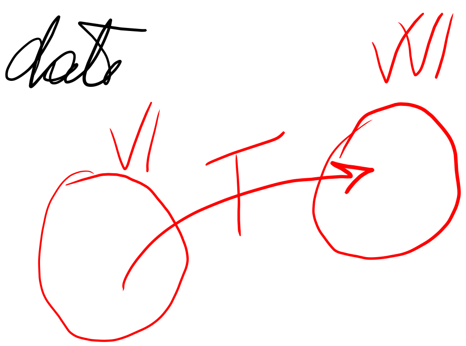
$v \in Nu(T)$

$$[v]^B = \begin{pmatrix}-\beta\\ \beta\\ -\beta\end{pmatrix} \Rightarrow v = -\beta v_1 + \beta v_2 - \beta v_3 = \beta\begin{pmatrix}0\\-1\\0\end{pmatrix}$$

$$y = T\begin{pmatrix}6\\6\\6\end{pmatrix} \qquad M_B^E\left[\begin{pmatrix}6\\6\\6\end{pmatrix}\right]^B = M_B^E\begin{pmatrix}6\\0\\6\end{pmatrix} = [T(x)]^E = y$$

$$M_B^E\,\underbrace{[x]^B}_{\text{infinitas soluciones}} = y$$

(4) $T: \mathbb{R}_3[x] \to \mathbb{R}_2[x]$ $\qquad [T]_B^E \qquad B$ dato



$$T^{-1}(S) \qquad S = \text{gen}\{1+x^2\}$$

$\dim V > \dim W$

$$[1+x^2]^E = \begin{pmatrix}1\\0\\1\end{pmatrix} \qquad [v]^B = \begin{pmatrix}\lambda - x_3\\0\\x_3\\0\end{pmatrix}$$

$$[T]_B^E\,[v]^B = \lambda\begin{pmatrix}1\\0\\1\end{pmatrix}$$

$$\begin{pmatrix}1 & 0 & 1 & 1\\0 & 1 & 0 & -1\\1 & 1 & 1 & 1\end{pmatrix}\begin{pmatrix}x_1\\x_2\\x_3\\x_4\end{pmatrix} = \begin{pmatrix}\lambda\\0\\\lambda\end{pmatrix}$$

Resolvemos:

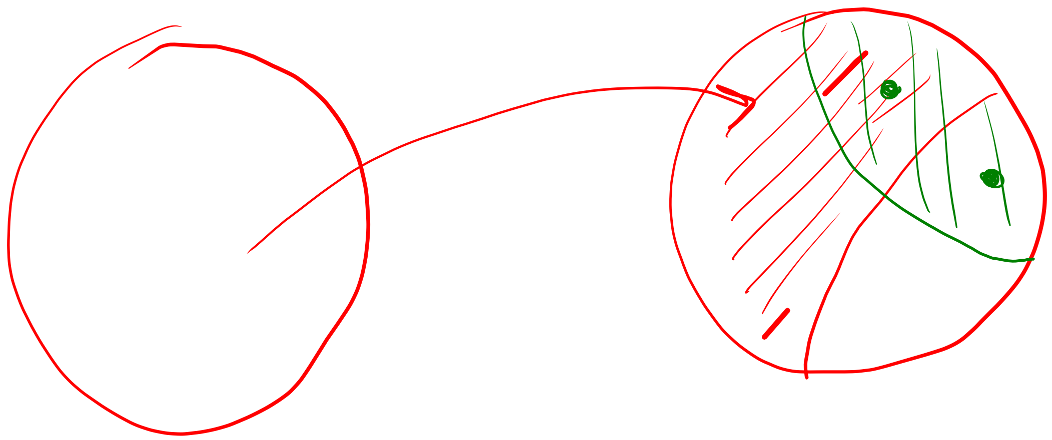
$$x_2 = x_4 = 0$$

$$x_1 + x_3 = \lambda$$

$$(\lambda - x_3)\cdot 1 + x_3(2+3x) = \lambda + x_3(1+3x)$$

Rta: $T^{-1}(S) = \text{gen}\{1 \,;\, 1+3x\}$

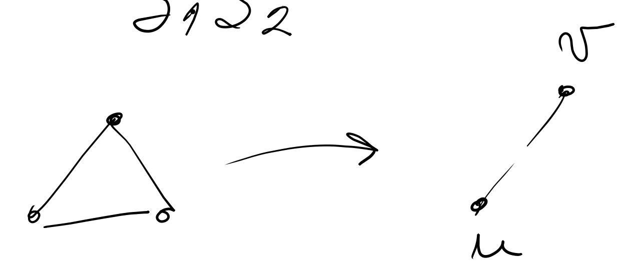
$$B = \{1,\ x^3 \,;\, 2+3x \,;\, 5x+8x^2\}$$



No es lo que pase en este ejercicio

5 Segments

$$S_1 = \left\{ \underset{v}{\begin{pmatrix}1\\1\\0\end{pmatrix}} \begin{pmatrix}0\\0\\1\end{pmatrix}_{u} \right\} \qquad S_2 = \left\{ \underset{w}{\begin{pmatrix}1\\-1\\0\end{pmatrix}} \right\} \qquad \Pi_{S_1 S_2}$$



$v_1 = v \longrightarrow v_1$

$v_2 = u \longrightarrow v_2$

$$v_3 = \begin{pmatrix}2\\0\\0\end{pmatrix} = \begin{pmatrix}1\\1\\0\end{pmatrix} + \begin{pmatrix}1\\-1\\0\end{pmatrix} = \underbrace{u}_{\in S_1} + \underbrace{w}_{\in S_2} \longrightarrow u$$

⑥ $\Sigma \circ \Sigma = I$ $\quad \Sigma$ simetría

$$\Sigma_{S_1 S_2}(v) = \Sigma_{S_1 S_2}(v_1 + v_2) = \underset{\in S_1}{v_1} - \underset{\in S_2}{v_2}$$

$$\Sigma\begin{pmatrix}4\\0\\-3\end{pmatrix} = \begin{pmatrix}0\\0\\1\end{pmatrix} \qquad \Sigma\begin{pmatrix}3\\1\\-2\end{pmatrix} = \begin{pmatrix}5\\1\\-4\end{pmatrix}$$

$$u_1 + u_2 \qquad u_1 - u_2 \qquad w_1 + w_2 \qquad w_1 - w_2$$

$$u_1 + u_2 = \begin{pmatrix}4\\0\\-3\end{pmatrix} \qquad w_1 + w_2 = \begin{pmatrix}3\\1\\-2\end{pmatrix}$$

$\oplus$ $2u_1$

$\ominus$ $2u_2$

$$u_1 - u_2 = \begin{pmatrix}0\\0\\1\end{pmatrix} \qquad w_1 - w_2 = \begin{pmatrix}5\\1\\-4\end{pmatrix}$$

$\{u_1, w_1\}$ LI

$\{u_2, w_2\}$ LD

$$u_1 = \begin{pmatrix}2\\0\\-1\end{pmatrix} \quad u_2 = \begin{pmatrix}2\\0\\-2\end{pmatrix} \quad w_1 = \begin{pmatrix}4\\1\\-3\end{pmatrix} \quad w_2 = \begin{pmatrix}-1\\0\\1\end{pmatrix}$$

$$\in S_1 \qquad \in S_2 \qquad \in S_1 \qquad \in S_2$$

$$u_1 = \begin{pmatrix} 2 \\ 0 \\ -1 \end{pmatrix} \in S_1 \qquad u_2 = \begin{pmatrix} 2 \\ 0 \\ -2 \end{pmatrix} \in S_2 \qquad w_1 = \begin{pmatrix} 4 \\ 1 \\ -3 \end{pmatrix} \in S_1 \qquad w_2 = \begin{pmatrix} -1 \\ 0 \\ 1 \end{pmatrix} \in S_2$$

$$S_1 = \text{gen}\left\{ \underbrace{\begin{pmatrix} 2 \\ 0 \\ -1 \end{pmatrix}}_{u_1} \underbrace{\begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix}}_{w_1 - 2u_1} \right\} \qquad S_2 = \text{gen}\left\{ \begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix} \right\}$$

$$\Sigma(S) \qquad S = \{ x \in \mathbb{R}^3 : x_2 = 0 \} = \text{gen}\left\{ \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \right\}$$

$$S_2 \subset S \qquad = \text{gen}\left\{ \begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix} \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \right\}$$

$$\Sigma \begin{pmatrix} 4 \\ 0 \\ -3 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \rightarrow \Sigma \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 4 \\ 0 \\ -3 \end{pmatrix} \qquad \Sigma \quad \begin{pmatrix} -1 \\ 0 \\ 1 \end{pmatrix} \quad \begin{pmatrix} 4 \\ 0 \\ -3 \end{pmatrix}$$

$$\Sigma(S) = \text{gen}\left\{ \begin{pmatrix} -1 \\ 0 \\ 1 \end{pmatrix} \begin{pmatrix} 4 \\ 0 \\ -3 \end{pmatrix} \right\}$$

1) $T: \mathbb{R}^3 \to \mathbb{R}_2[x]$

$$B = \left\{ \begin{array}{ccc} v_1 & v_2 & v_3 \\ \begin{pmatrix}0\\0\\1\end{pmatrix} & \begin{pmatrix}0\\1\\1\end{pmatrix} & \begin{pmatrix}1\\1\\1\end{pmatrix} \end{array} \right\}$$

$$C = \left\{ \begin{array}{ccc} w_1 & w_2 & w_3 \\ 1, & 1+x, & 1+x+x^2 \end{array} \right\}$$

$$[T]_B^C = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 1 & -1 \end{pmatrix}$$

$$[T(v_1)]^C \Rightarrow v_1 \in Nu(T) = gen\left\{ \begin{pmatrix}0\\0\\1\end{pmatrix} \right\}$$

$$Im(T) = gen\{ w_1 + w_2 + w_3 \, ; \, -w_3 \}$$

$$Base\ Nu(T) = \left\{ \begin{pmatrix}0\\0\\1\end{pmatrix} \right\} \qquad Base\ Im(T) = \{ 3+2x+x^2 \, ; \, 1+x+x^2 \}$$

OJO con las coordenadas y lo que son realmente vectores de V, W, Nu(T), Im(T)

2) $V = \{A \in \mathbb{R}^{2\times 2} : A = A^T\}$ $\quad T : \underbrace{\mathbb{R}_2[x]} \to V$

$\dim \mathbb{R}_2[x] = 3$

$$T(p) = \begin{pmatrix} p'(0) & p(0) \\ p(0) & 0 \end{pmatrix}$$

$\mathrm{Nu}(T) = \{ p / : T(p) = 0_V \}$

$p = a + bx + cx^2$

$p(0) = a = 0$

$p'(0) = b = 0$

a) Pruebe que T es lineal

$T(\alpha p + q) = \alpha T(p) + T(q)$

$$T(\alpha p + q) = \begin{pmatrix} (\alpha p + q)'(0) & (\alpha p + q)(0) \\ (\alpha p + q)(0) & 0 \end{pmatrix} =$$

$$= \begin{pmatrix} \alpha p'(0) + q'(0) & \alpha p(0) + q(0) \\ \alpha p(0) + q(0) & 0 \end{pmatrix} =$$

$\mathrm{Nu}(T) = \mathrm{gen}\{x^2\}$

$$= \alpha T(p) + T(q)$$

$Im(T)$

$$T(p) = \begin{pmatrix} p'(0) & p(0) \\ p(0) & 0 \end{pmatrix} = \begin{pmatrix} b & a \\ a & 0 \end{pmatrix} =$$

$$p = a + bx + cx^2 \qquad b\begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} + a\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

$$\Rightarrow Im(T) = gen\left\{ \underbrace{\begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}}_{A_1} \underbrace{\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}}_{A_2} \right\}$$

$$Base_{Im(T)} = \{A_1, A_2\} \qquad Base_{Nu(T)} = \{x^2\}$$

Sea $A = \begin{pmatrix} 0 & 1 \\ 1 & a \end{pmatrix}$ A es simétrica $\Rightarrow A \in V$

$T^{-1}(\{A\}) \neq \emptyset \Rightarrow A \in \operatorname{Im}(T) \Rightarrow \boxed{a = 0}$