Ejemplo $\langle x ; y\rangle = y^T \begin{pmatrix} 2 & 1 \\ 1 & 3 \end{pmatrix} x$ $\quad 2>0 \quad 1>0$

$G_E$ $\qquad \det G = 1 > 0$

Matriz de producto interno

$2 = \left\langle \begin{pmatrix} 1 \\ 0 \end{pmatrix} \begin{pmatrix} 1 \\ 0 \end{pmatrix} \right\rangle \qquad 1 = \left\langle \begin{pmatrix} 1 \\ 0 \end{pmatrix} \begin{pmatrix} 0 \\ 1 \end{pmatrix} \right\rangle = \left\langle \begin{pmatrix} 0 \\ 1 \end{pmatrix} \begin{pmatrix} 1 \\ 0 \end{pmatrix} \right\rangle \qquad 3 = \left\langle \begin{pmatrix} 0 \\ 1 \end{pmatrix} \begin{pmatrix} 0 \\ 1 \end{pmatrix} \right\rangle$

FÓRMULA $\langle x, y\rangle = \begin{pmatrix} y_1 & y_2 \end{pmatrix} \begin{pmatrix} 2 & 1 \\ 1 & 3 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} =$

$(2y_1 + y_2)x_1 + (y_1 + 3y_2)x_2 = 2y_1x_1 + 1x_1y_2 + 1x_2y_1 + 3x_2y_2$

$\left\langle \begin{pmatrix} 1 \\ 2 \end{pmatrix} \begin{pmatrix} 0 \\ -1 \end{pmatrix} \right\rangle = \left[\begin{pmatrix} 0 \\ -1 \end{pmatrix}\right]^{E\,T} G_E \left[\begin{pmatrix} 1 \\ 2 \end{pmatrix}\right]^E = \begin{pmatrix} 0 & -1 \end{pmatrix} \begin{pmatrix} 2 & 1 \\ 1 & 3 \end{pmatrix} \begin{pmatrix} 1 \\ 2 \end{pmatrix} = -1-6$

$= -7$

Con la formula

$\langle x, y\rangle = \left\langle \begin{pmatrix} 1 \\ 2 \end{pmatrix} \begin{pmatrix} 0 \\ -1 \end{pmatrix} \right\rangle = 2\ 1 . 0 + 1 . 1(-1) + 1 . 0 . 2 + 3\ 2(-1) = -7$

$$\langle x, y\rangle = 2 y_1 x_1 + 1 x_1 y_2 + 1 x_2 y_1 + 3 x_2 y_2$$

$$\mathcal{B} = \left\{ \underset{v_1}{\begin{pmatrix} 1 \\ 1 \end{pmatrix}}, \underset{v_2}{\begin{pmatrix} 2 \\ 1 \end{pmatrix}} \right\} \qquad G_{\mathcal{B}} = \begin{pmatrix} \|v_1\|^2 & \langle v_1, v_2\rangle \\ \langle v_1, v_2\rangle & \|v_2\|^2 \end{pmatrix} = \begin{pmatrix} 7 & 10 \\ 10 & 15 \end{pmatrix}$$

$$\left\langle \begin{pmatrix} 1 \\ 1 \end{pmatrix} \begin{pmatrix} 1 \\ 1 \end{pmatrix} \right\rangle = 2.1\,1 + 1.1.1 + 1.1.1 + 3.\,1.1 = 7$$

$$\left\langle \begin{pmatrix} 1 \\ 1 \end{pmatrix} \begin{pmatrix} 2 \\ 1 \end{pmatrix} \right\rangle = 2.2\,1 + 1.1\,1 + 1.1\,2 + 3.1.1 = 10$$

$$\left\langle \begin{pmatrix} 2 \\ 1 \end{pmatrix} \begin{pmatrix} 2 \\ 1 \end{pmatrix} \right\rangle = 2.2.2 + 1.2\,1 + 1.1.2 + 3.1\,1 = 15$$

$x_1 = 2 = y_1 \qquad x_2 = 1 = y_2$

$$\left\langle \begin{pmatrix} 1 \\ 2 \end{pmatrix} \begin{pmatrix} 0 \\ -1 \end{pmatrix} \right\rangle = \begin{pmatrix} -2 & 1 \end{pmatrix} \begin{pmatrix} 7 & 10 \\ 10 & 15 \end{pmatrix} \begin{pmatrix} 3 \\ -1 \end{pmatrix} = -4.3 + 5 = -7$$

$$\begin{pmatrix} 1 \\ 2 \end{pmatrix} = \alpha v_1 + \beta v_2 = \alpha \begin{pmatrix} 1 \\ 1 \end{pmatrix} + \beta \begin{pmatrix} 2 \\ 1 \end{pmatrix} \qquad \begin{pmatrix} 0 \\ -1 \end{pmatrix} = \alpha \begin{pmatrix} 1 \\ 1 \end{pmatrix} + \beta \begin{pmatrix} 2 \\ 1 \end{pmatrix}$$

$$\begin{cases} \alpha + 2\beta = 1 \\ \alpha + \beta = 2 \end{cases} \quad \beta = -1 \quad \alpha = 3 \qquad \begin{cases} \alpha + 2\beta = 0 \\ \alpha + \beta = -1 \end{cases} \quad \beta = 1 \quad \alpha = -2$$

$$\left[\begin{pmatrix} 1 \\ 2 \end{pmatrix}\right]^{\mathcal{B}} = \begin{pmatrix} 3 \\ -1 \end{pmatrix} \qquad \left[\begin{pmatrix} 0 \\ -1 \end{pmatrix}\right]^{\mathcal{B}} = \begin{pmatrix} -2 \\ 1 \end{pmatrix}$$

$\mathbb{R}^3$ pie $\quad A = \left\{ \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} \right\} \quad v = \begin{pmatrix} x \\ y \\ z \end{pmatrix}$

$A^{\perp} = \left\{ v \in \mathbb{R}^3 : \langle v , \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} \rangle = 0 \right\} \Rightarrow x + y = 0$

$A^{\perp} = \{ v \in \mathbb{R}^3 : x + y = 0 \}$

$(A^{\perp})^{\perp} = \{ u \in \mathbb{R}^3 : \langle u, v \rangle = 0 \quad \forall v \in A^{\perp} \}$

¿Cuántos elementos hay en $A^{\perp}$? $\longrightarrow$ Infinitos

¿Tengo que hacer infinitas cuentas? No

Si considero un S. generador de $A^{\perp} = \text{gen} \left\{ \begin{pmatrix} 1 \\ -1 \\ 0 \end{pmatrix} \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \right\}$

y busco $\left\{ \begin{pmatrix} 1 \\ -1 \\ 0 \end{pmatrix} \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \right\}^{\perp} = \text{gen} \left\{ \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} \right\}$

$u = \alpha \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} \perp \begin{pmatrix} 1 \\ -1 \\ 0 \end{pmatrix} \perp \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \Rightarrow (A^{\perp})^{\perp} = \text{gen}\{A\}$